Simple Far-Field Model for Lateral Leakage in Printed Transmission Lines

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ABSTRACT — **A simple far-field phased-line model for the lateral leakage (e.g. the surface-wave one) in printed transmission lines is presented. It is based on replacing the guiding strip(s) and/or guiding slot(s) by a phased line radiating in the background structure (usually a simple dielectric-slab guide or a partially filled parallel plate waveguide). Both full-line (infinitely extended) and half-line (semi-infinite) structures are considered. The model predicts the impossibility of real (bounded) modes that are faster than the corresponding background mode (into which leakage can take place) in both full-line and half-line structures. On the other hand, complex (leaky-wave) modes are shown to behave properly (improperly) outside their sector of definition in half-line (full-line) structures. Far-field distributions for a number of cases are given for the sake of illustration.**

I. Introduction

The study of lateral leakage in printed transmission lines and periodic structures has attracted the interest of many researchers in the last two decades (see e.g. [1]-[4]). This has been mainly due to the associated power loss and cross-talk problems that severely degrade the performance of high density MIC and MMIC modules.

Two of the well known facts related to the leakage issue are: 1. The impossibility of real (bounded) modes that are faster than the corresponding background mode (in which leakage can take place). 2. The improper (non-physical) behavior of complex (leaky-wave) modes outside their sector of definition. Such fundamental properties have been rigorously proved for infinitely extended open guiding structures for both space-wave leakage (e.g. [5]-[7]) and surface-wave one [e.g. [1]-[2]). On the other hand, the excitation of semi-infinite structures has been investigated only recently [8]. It has been shown there that leaky-wave modes excited in some semi-infinite open guiding structures behave properly both inside and outside their sector of definition. This has been explained as being due to the interference with the field generated by the additional source associated with the step-function behavior of the semi-infinite structure.

In this contribution, we present a simple model for the far-field behavior of the real (bounded) and complex (leaky-wave) modes in both infinite and semi-infinite printed transmission lines. The model is based on replacing the guiding strip(s) and/or slot(s) by a phased line whose current has the same propagation constant as that of the mode under consideration. The background structure remains however unchanged. In order to avoid mixing up lateral (surface-wave) and space-wave leakage, only covered structures will be considered. The corresponding background structure is either homogeneously or partially filled parallel plate waveguide (PPW). Extending the validity of the model to uncovered structures is however straightforward.

II. Basic Formulation

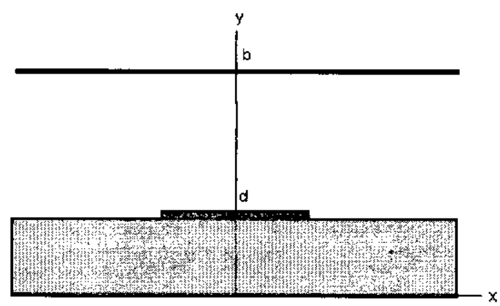

Fig.1: A covered microstrip line

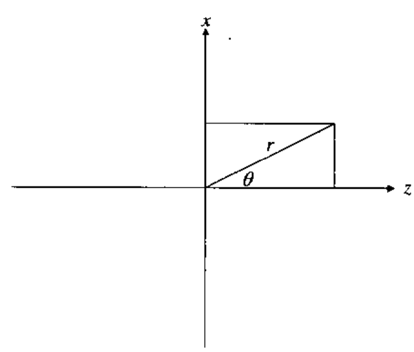

Fig. 2: Cartesian and polar coordinates

Let us consider the covered microstrip line shown in Fig. 1 as representative for laterally open structures. Referring to Fig. 2, except for a very small sector in the vicinity of θ=0, the strip thickness can be neglected in a far-field analysis of the structure. The strip can then be replaced by an infinitesimally thin phased line whose current density is given by:

$$J_z(x,y,z) = \delta(x)\delta(y-d)e^{-j\beta z}\begin{cases} 1 & (full-line) \\ u(z) & (half-line) \end{cases} \quad (1)$$

where β is the propagation constant of the mode under consideration and $u(z)$ is the unit step function. The Green's function of the background structure (the partially filled PPW) with source and field (observation) point coordinates (r_0, θ_0, y_0) and (r, θ, y), respectively, is given by (see e.g.[7]):

$$G(r,\theta,y;r_0,\theta_0,y_0) = \frac{1}{j2b}\sum_{n=1}^{\infty} v_n(y)v_n(y_0)V_n(r,\theta;r_0,\theta_0) \quad (2)$$

where $v_n(y)$ has a sinusoidal dependence within the dielectric region and either exponential (for the first few values of n) or sinusoidal (for the higher values of n) in the air region and

$$V_n(r,\theta;r_0,\theta_0) = \sum_{m=-\infty}^{\infty} e^{jm(\theta-\theta_0)} J_m(k_n r_<) H_m^{(2)}(k_n r_>) \quad (3)$$

with $r_<$ and $r_>$ being the smaller and larger value of (r, r_0), respectively, J_m and $H_m^{(2)}$ being the Bessel and outgoing Hankel function or order m, respectively, and k_n is the propagation constant of the n^{th} mode in the background structure. The latter is real (imaginary) when the corresponding background mode is propagating (evanescent).

A typical modal field component is readily shown to be given by:

$$F(r,\theta,y;\beta) = \frac{1}{j2b}\sum_{n=1}^{\infty} v_n(y)v_n(d)I_n(r,\theta;\beta) \quad (4\text{-a})$$

$$I_n(r,\theta;\beta) = \sum_{m=-\infty}^{\infty} e^{jm\theta}\left(I_{nm}^{(1)}(r;\beta) + I_{nm}^{(2)}(r;\beta)\right) \quad (4\text{-b})$$

$$I_{nm}^{(1)}(r;\beta) = H_m^{(2)}(k_n r)\int_0^{\infty} J_m(k_n r_0)K_m(r_0;\beta)dr_0 \quad (4\text{-c})$$

$$I_{nm}^{(2)}(r;\beta) = \int_0^{\infty}\left(H_m^{(2)}(k_n r_0)J_m(k_n r) - J_m(k_n r_0)H_m^{(2)}(k_n r)\right)K_m(r_0;\beta)dr_0 \quad (4\text{-d})$$

$$K_m(r_0;\beta) = \begin{cases} e^{-j\beta r_0} & (half-line) \\ (e^{-j\beta r_0} + e^{-jm\pi}e^{j\beta r_0}) & (full-line) \end{cases} \quad (4\text{-e})$$

It is readily shown that $I_{nm}^{(2)}(r;\beta)$ is given in terms of the Fresnel integrals [9] and $I_{nm}^{(2)}(r;\beta) \rightarrow (k_n r)^{-1}$ as $k_n r \rightarrow \infty$. In the subsequent analysis, we will consider the possibility of leakage into the n^{th} background mode. This means that the operating frequency is chosen such that this mode is propagating and hence k_n is real.

A. Real Modes in Semi-Infinite (Half-Line) Structures

The integral in (4-c) is readily shown to reduce to [9]:

$$I_{nm}^{(1)}(r;\beta) = e^{-jm\frac{\pi}{2}} H_m^{(2)}(k_n r)\frac{e^{jm\phi}}{k_n \sin\phi} \quad (5\text{-a})$$

$$\varphi = \cos^{-1}\left(\frac{\beta}{k_n}\right) = \begin{cases} \varphi_0,\ (0 < \varphi_0 \le \frac{\pi}{2}) & (\beta < k_n) \\ j\eta_0,\ (\eta_0 > 0) & (\beta > k_n) \end{cases} \quad (5\text{-b})$$

For a far-field consideration, $k_n r \rightarrow \infty$ and hence

$$I_{nm}^{(1)}(r;\beta) \rightarrow \sqrt{\frac{2}{\pi k_n r}}e^{-j(k_n r - \frac{\pi}{4})}\frac{e^{jm\phi}}{k_n \sin\phi} \quad (6)$$

$I_{nm}^{(2)}(r;\beta)$ can then be neglected with respect to $I_{nm}^{(1)}(r;\beta)$ and (4-b) reduces to

$$I_n(r,\theta;\beta) \rightarrow \sqrt{\frac{2}{\pi k_n r}}e^{-j(k_n r - \frac{\pi}{4})}\frac{1}{k_n \sin\phi}\left(1 + 2\sum_{m=1}^{\infty} e^{jm\phi}\cos m\theta\right) \quad (7)$$

The above series has a Dirac-delta behavior at $\theta = \pm\phi_0$ for $\beta < k_n$ which leads to an infinite radiated power and hence to the non-physicality of real modes with $\beta < k_n$. On the other hand, for $\beta > k_n$, (7) converges uniformly to:

$$I_n(r,\theta;\beta) \rightarrow \sqrt{\frac{2}{\pi k_n r}}e^{-j(k_n r - \frac{\pi}{4})}\frac{2e^{-\eta_0}}{jk_n(1 + e^{-2\eta_0} - 2e^{-\eta_0}\cos\theta)} \quad (8)$$

Fig. 3 shows the angular dependence of $|I_n(r,\theta;\beta)|$ given by (8) for different values of η_0.

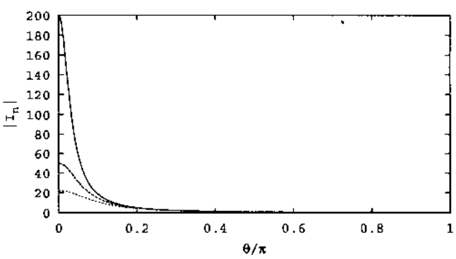

Fig. 3: $|I_n|$ (arbitrary unit) for η_0=0.1 (solid line), η_0=0.2 (----) and η_0=0.3 (- - -)

B. Real Modes in Infinite (Full-Line) Structures

For this case, $I_{nm}^{(1)}(r;\beta)$ reduces to:

$$I_{nm}^{(1)}(r;\beta) = \begin{cases} 2e^{-jm\frac{\pi}{2}} H_m^{(2)}(k_n r)\frac{\cos m\phi_0}{k_n \sin\phi_0} & (\beta < k_n) \\ 0 & (\beta > k_n) \end{cases} \quad (9)$$

For a far-field consideration, $I_{nm}^{(1)}(r;\beta) \to (k_n r)^{\frac{-1}{2}}$ for $\beta < k_n$, while $I_{nm}^{(2)}(r;\beta) \to (k_n r)^{-1}$ for both $\beta < k_n$ and $\beta > k_n$. So, similar to the half-line structure, $I_n(r,\theta;\beta)$ of the full-line structure diverges at $\theta = \pm\phi_0$ for $\beta < k_n$ and converges uniformly to a physical radiation field (with a radial dependence $(k_n r)^{-1}$) for $\beta > k_n$.

It is worth noting here that as η_0 increases, $I_n(r,\theta;\beta)$ in (8) tends very rapidly to zero and becomes negligible with respect to that obtained by replacing $I_{nm}^{(1)}(r;\beta)$ by $I_{nm}^{(2)}(r;\beta)$. The latter converges uniformly to $(k_n r)^{-1}$ as $k_n r \to \infty$. Consequently, the far-field of both half-line and full-line structures behaves similarly for real modes with $\beta > k_n$, except for $\beta \cong k_n$ ($\eta_0 \cong 0$).

C. Leaky-Wave Modes in Semi-Infinite Structures

Th propagation constant of a leaky-wave mode is complex with a positive real part and a negative imaginary part: $\beta = \beta' - j\beta''$. Equation (5-a) is valid for this case too, however with a complex value of ϕ:

$$\phi = \cos^{-1}\left(\frac{\beta}{k_n}\right) = \phi_0 + j\eta_0, \quad 0 < \phi_0 \le \frac{\pi}{2}, \quad \eta_0 > 0 \tag{10}$$

A far-field consideration leads to $I_{nm}^{(1)}(r;\beta)$ given by (6). Again, $I_{nm}^{(2)}(r;\beta)$ (which has an asymptotic value $(k_n r)^{-1}$) can be neglected with respect to $I_{nm}^{(1)}(r;\beta)$ resulting in $I_n(r,\theta;\beta)$ given by (7). Due to the positive value of η_0, the series in (7) converges uniformly *for all values of θ*.

$$I_n(r,\theta;\beta) \to \sqrt{\frac{2}{\pi k_n r}}\,\frac{e^{-j(k_n r - \frac{\pi}{4})}}{k_n \sin\phi}\left(\frac{1}{1-\lambda e^{j(\phi_0+\theta)}} + \frac{1}{1-\lambda e^{j(\phi_0-\theta)}} - 1\right) \tag{11}$$

where $\lambda = e^{-\eta_0} < 1$. Such a convergent behavior contradicts the well known fact that leaky-wave modes converge within their sector of definition only (here for $0 \le \theta < \phi_0$) [5]. A similar behavior has been however observed recently in [8] using a full-wave near-field analysis. It has been explained there as being due to the effect of the step-function source accompanying any semi-infinite structure. Fig. 4 shows the angular dependence of $|I_n(r,\theta;\beta)|$ given by (11).

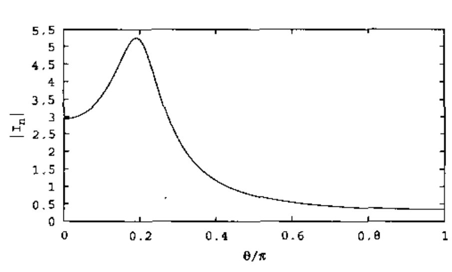

Fig. 4: $|I_n|$ (arbitrary unit) for η_0=0.2 and ϕ_0=0.2π

The field is maximum at $\theta = \phi_0$. It is worth noting that the angle of maximum field ϕ_0 is given by $\phi_0 = \Re e\{\cos^{-1}(\beta / k_n)\}$ and not by $\phi_0 = \cos^{-1}(\Re e\{\beta\}/k_n)$. The latter expression has been always taken in the literature as an approximation for weak leakage. The two expressions are in fact very near if $\beta'' = -\Im m\{\beta\} << \beta' = \Re e\{\beta\}$ which characterizes weak leakage.

D. Leaky-Wave Modes in Infinite Structures

The field excited by the *negative* half-line (which extends over the negative values of z_0) is easily shown to modify $I_{nm}^{(1)}(r;\beta)$ into:

$$I_{nm}^{(1)}(r;\beta) = 2e^{-jm\frac{\pi}{2}} H_m^{(2)}(k_n r)\frac{\cos m\phi}{k_n \sin\phi} \tag{12}$$

Again, for a far-field consideration, $I_{nm}^{(2)}(r;\beta)$ can be neglected with respect to $I_{nm}^{(1)}(r;\beta)$ and the following expression is obtained for $I_n(r,\theta;\beta)$:

$$I_n(r,\theta;\beta) = \frac{2}{k_n \sin\phi}\left(H_0^{(2)}(k_n r) + 2\sum_{m=1}^{\infty} e^{-jm\frac{\pi}{2}} H_m^{(2)}(k_n r)\cos m\phi \cos m\theta\right) \tag{13}$$

Replacing $H_m^{(2)}(k_n r)$ by their asymptotic values for $k_n r \to \infty$ results in a series which diverges due to the terms $\cosh m\eta_0$ and $\sinh m\eta_0$. We have tried to isolate the source of divergence into a term, which is familiar for leaky-wave modes. So, after some mathematical manipulations, the asymptotic value of $I_n(r,\theta;\beta)$ in (13) can be rewritten as:

$$I_n(r,\theta;\beta) \to \frac{\left(e^{-j(\beta z + k_x x)}(1+\cos(\theta-\phi)) + e^{-j(\beta z - k_x x)}(1+\cos(\theta+\phi))\right)}{k_n \sin\phi} \tag{14}$$

where $k_x = k_n \sin\phi$. The above expression resembles the familiar behavior of leaky-wave modes. It converges within the sector $\beta'' z > |\Im m\{k_x\} x|$ only.

Comparing the leakage behavior in infinite and semi-infinite structures, it is easily seen that the negative half-line only is responsible for the non-physicality of leaky-wave modes outside their sector of definition. This is however easily explained by the fact that the current of the negative half-line increases unlimitedly as $z_0 \to -\infty$.

III. Conclusion

A simple far-field model for lateral leakage in printed transmission lines has been presented. The model has been applied to describe leakage in both semi-infinite and infinite structures. It predicts the impossibility of real fast-wave propagation in both semi-infinite and infinite structures. On the other hand, real slow-wave propagation behaves similarly in both structures. The model justify the recently published results that complex leaky-wave propagation converges everywhere in semi-infinite structures. The divergence of leaky-wave modes outside their sector of definition in infinite structures is predicted by the model as well.

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